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A Gaussian-distributed target evasion region for rectangular coordinates is developed by employing a Rayleigh density function to model evasion speed changes and a uniform density function to model evasion course changes. The upper, infinite tail of the Rayleigh density is truncated at maximum target speed, resulting in a conditional Gaussian density function model of the target evasion region. The mean and variance for this positional density function are formulated to aid in determining new firing strategies based on evasion tactics. Selecting large values for the mode of the truncated Rayleigh density function allows for comparison with a ramp density function. The results show that, for this case, the truncated Gaussian density function approximates a uniform density function.

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Development of a Target Evasion Region Having a Conditional Gaussian Distribution

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PREFACE

This study was conducted under NUSC's Independent Research Program, Project No. B30525, "Weapon Time-of-Fire Performance Prediction," principal investigator J.J. Perruzzi (Code 2221).

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REVIEWED AND APPROVED: 1 JULY 1987

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I. INTRODUCTION

In an earlier report (reference 1), a generalized probability density function was developed for modeling maneuvering target position inside a bounded, time-varying, evasion region (reference 2). The approach taken was to assume beta density functions for changes in evasion speed and course, and then formulate the resultant positional density function. Beta density functions were employed because of their flexibility; i.e., uniform, ramp, or other type density functions can be easily developed via proper selection of the shaping parameters.

One reason for developing these probabilistic evasion regions was to formulate alternate firing strategies for advanced weapons. The mean or mode of the density function can serve as an aimpoint based on target evasion. Another use for these density functions is in the area of online measure-of-effectiveness (MOE) algorithms. MOE algorithms allow real-time prediction of weapon performance in a tactical encounter. To be of value, these algorithms must take into account the initial target motion analysis (TMA) uncertainty region, the target evasion region, weapon delivery errors, and weapon destruct characteristics.

The work presented in reference 1 provides an analytical formulation of the evasion statistics; but these statistics must be combined with the TMA uncertainty region, which has Gaussian characteristics. This results in a total target uncertainty/evasion region density function. The combined region is very difficult to determine analytically because of the complexity of the evasion region density functions. However, if it is assumed that both the TMA uncertainty region and the evasion region are Gaussian distributed, then the two densities could very easily be combined to form another Gaussian density function (reference 3). Moreover, this combined density function would contain all of the pertinent statistical information inherent in the TMA uncertainty and the evasion region densities.

This report documents development of a Gaussian-distributed target evasion region. To obtain such a region, target evasion speed is modeled by a Rayleigh distribution, while target evasion course uses a uniform density function. Although the uniform course/Rayleigh speed model yields a positional density function that is Gaussian distributed, there is a disadvantage in using the Rayleigh density function because of its infinite tail. Since maximum evasion speed (S_{max}) is finite, the modeling parameter in the Rayleigh density that determines the most likely evasion speed must be less than S_{max}. In fact, it has been shown (reference 4) that this modeling

parameter should be less than 0.41 S_{max} to ensure that there is less than a 5 percent chance of obtaining evasion speeds greater than S_{max}. To overcome this constraint, a conditional Rayleigh distribution must be used. The resulting positional uncertainty region then becomes a conditional Gaussian density function. Finally, it is shown that a beta density can be employed to approximate a conditional Rayleigh distribution for large values of the Rayleigh modeling parameter.

II. EVADING TARGET POSITIONAL DENSITY FUNCTIONS

Two analytical density functions that model the evasion region of a target were determined, along with their respective means and variances. The first positional density was formulated using uniform course/Rayleigh speed density functions as inputs. The second was developed for a uniform course/truncated Rayleigh speed combination. In this section, the analytical formulations of the models are presented and evaluated.

UNIFORM COURSE/RAYLEIGH SPEED COMBINATION

The form of a Rayleigh density function to model evasion speed is given by (reference 5)

$$f_S(S) = \frac{S \exp\{-S^2/(2\alpha^2)\}}{\alpha^2}, \quad 0 \leq S \leq \infty, \tag{1}$$

where S is the target speed. The modeling parameter α, which determines the mode of the density, would represent the most likely evasion speed. In addition, this parameter appears in the mean value of the density, which is written as (reference 5)

$$m_S = \frac{\alpha\sqrt{\pi}}{\sqrt{2}}. \tag{2}$$

Figure 1 shows a Rayleigh density function for modeling target evasion speed. The mode of the density occurs at α, which was set to a value of 10. Included in this illustration are the probabilities of obtaining evasion speeds greater than S_{max} (maximum target speed) for S_{max} values of 25, 30, and 40 knots. As can be seen, even for a maximum speed of 25 knots, the probability of exceeding this value is low. But α represents the most likely evasion speed, which is 10 knots in this example. For large values of α, the probability of obtaining evasion speeds greater than S_{max} increases. This probability is determined from

$$P(S \geq S_{max}) = \exp\{-S_{max}^2/(2\alpha^2)\}. \tag{3}$$

Since the target evasion regions are given as a function of time, changes in target speed can be scaled into changes in distances by

$$r = st, \tag{4}$$

where t is a deterministic quantity representing evasion time (growth of the evasion region (reference 2)). Now, the density function for speed changes can be transformed into an evasion range density function of the form

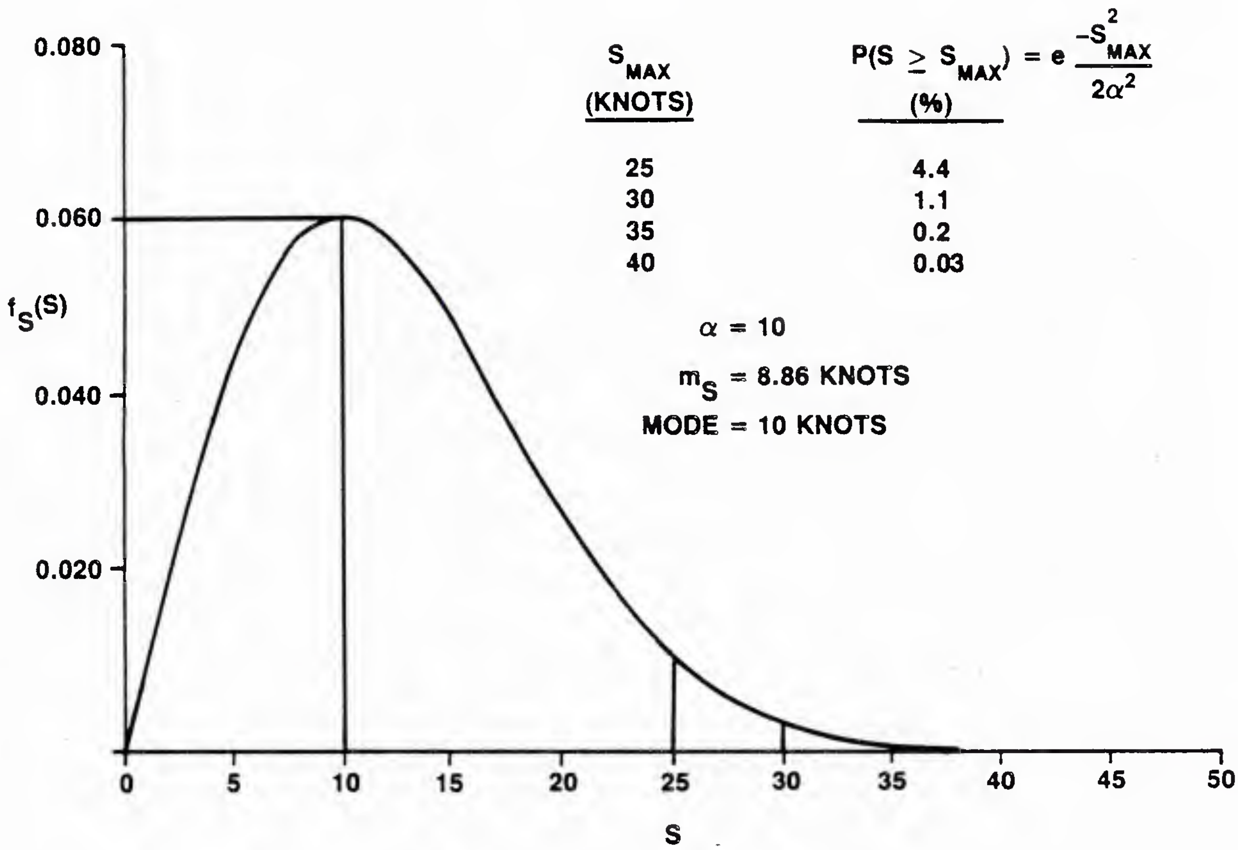

Figure 1. Rayleigh Density Function (α = 10)

$$f_r(r) = \frac{r\ \exp\{-r^2/(2\alpha^2 t^2)\}}{\alpha^2 t^2}, \quad 0 \leq r \leq \infty, \tag{5a}$$

and

$$m_r = \frac{\alpha t\sqrt{\pi}}{\sqrt{2}}. \tag{5b}$$

For the evasion course θ, a uniform density of the form

$$f_\theta(\theta) = 1/2\pi, \quad -\pi \leq \theta \leq \pi, \tag{6}$$

is used. Since $f_r(r)$ and $f_\theta(\theta)$ are independent, the joint density $f_{r\theta}(r,\theta)$ becomes

$$f_{r\theta}(r,\theta) = \frac{r\ \exp\{-r^2/(2\alpha^2 t^2)\}}{2\pi a^2 t^2}, \quad 0 \leq r \leq \infty, \quad -\pi \leq \theta \leq \pi. \tag{7}$$

To transform equation (7) from polar to rectangular coordinates, $f_{r\theta}(r,\theta)$ is multiplied by the Jacobian (reference 5), which is given by (reference 1)

$$J = 1\sqrt{x^2 + y^2} \ . \tag{8}$$

Transforming the joint density function into rectangular coordinates yields

$$f_{xy}(x,y) = \frac{\exp\{-(x^2 + y^2)/(2\alpha^2 t^2)\}}{2\pi\alpha^2 t^2} \ , \tag{9a}$$

which is a joint Gaussian probability density function with variance

$$\sigma_x^2 = \sigma_y^2 = (\alpha t)^2. \tag{9b}$$

Equation (9a) is an analytical description of the statistics of the target position based on the evasion time t and modeling parameter α. This type of evasion strategy can be easily combined with the initial Gaussian uncertainty region obtained from TMA techniques. An analytical formulation for this combined uncertainty region is a critical element in any MOE algorithm for realistic quantification of weapon performance.

From equation (9b), it is seen that the variance for the positional density function is determined from the parameter αt, which also determines the mode and mean of the Rayleigh density given by equation (5a). As was shown in figure 1, α must be small to minimize the probability of obtaining evasion speeds greater than S_{max}. This requirement results in a narrow positional density function to describe the evasion region (see the three-dimensional plot in figure 2). As α increases, the variance given by equation (9b) increases, causing the positional density function to spread out. Figures 3, 4, and 5 are plots of the positional density functions for α = 15, 20, and 30, respectively. Also contained in these figures is the probability of obtaining speeds greater than S_{max}N As can be seen, even for α = 15, there is a high probability of obtaining evasion speeds greater than S_{max}.

If an instantaneous motion model (course and speed changes are step functions) is used to update evading target position, then the evasion region would be a circle centered at the initial alerted target position with a radius given by

$$r_{max} = S_{max}t. \tag{10}$$

For large values of α, the resultant positional density function for this evasion region assigns a non-zero probability to points outside the circle of

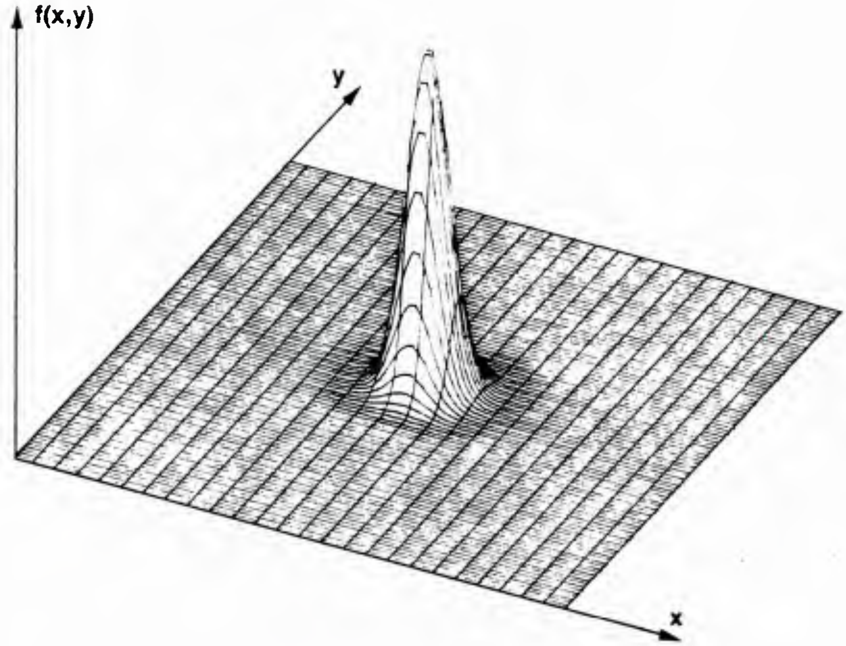

Figure 2. Three-Dimensional Gaussian Positional Density Function (α = 10)

radius r_{max}. This becomes apparent by examining the variance of the positional density function given by equation (9b). The three-sigma region is $3\alpha t$ and, if $\alpha >> S_{max}/3$, the three-sigma region would be larger than r_{max}. Accordingly, a positional probability region that is Gaussian distributed can only model targets evading at slow evasion speeds. Otherwise, such a model does not accurately depict the real evasion region.

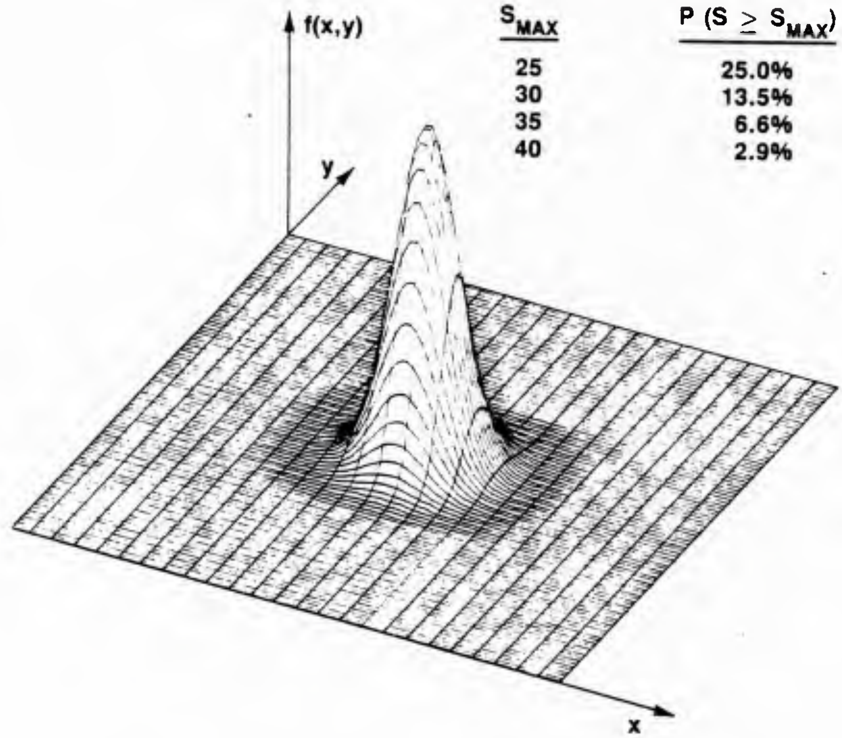

Figure 3. Three-Dimensional Gaussian Positional Density Function (α = 15)

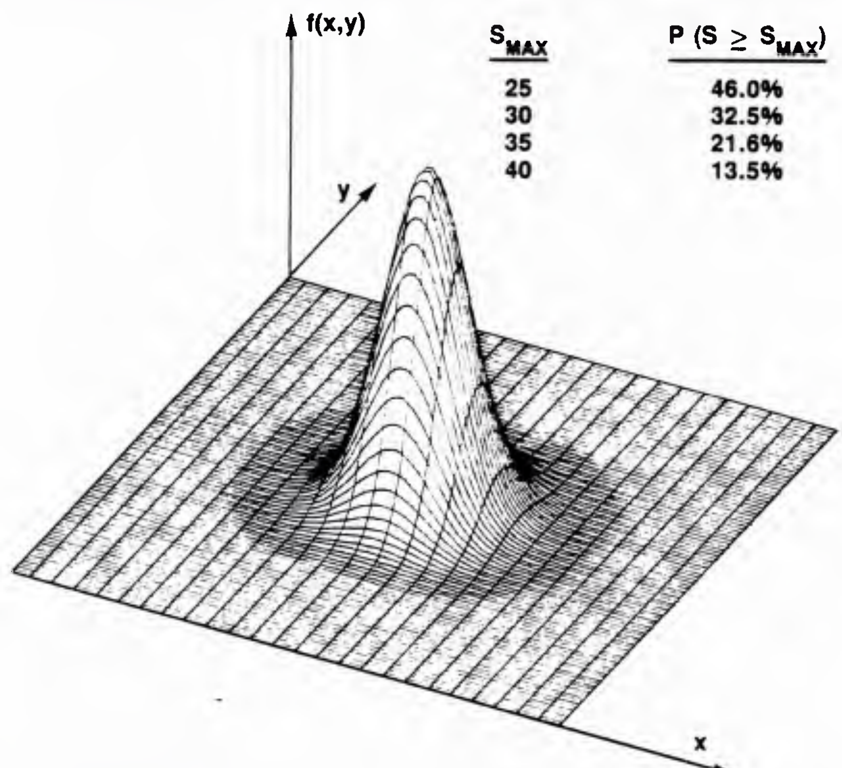

Figure 4. Three-Dimensional Gaussian Positional Density Function (α = 20)

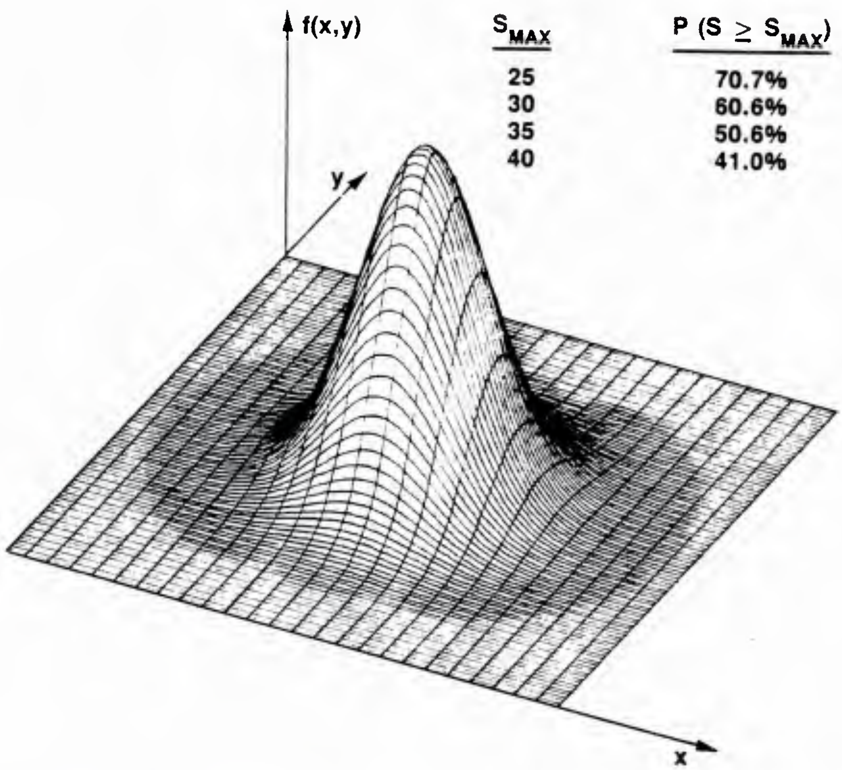

Figure 5. Three-Dimensional Gaussian Positional Density Function (α = 30)

CONDITIONAL RAYLEIGH DENSITY FUNCTION

Because of the restriction placed on the evasion speed density function in the preceding section, the resultant Gaussian uncertainty region can be used only for targets evading at slow speeds. To overcome this limitation and still obtain a Gaussian evasion region, a truncated or conditional distribution must be employed. This density is found from (reference 5)

$$f_S(S|S \leq S_{max}) = \frac{f_S(S)}{\int_0^{S_{max}} f_S(S)dS} , \tag{11}$$

where S_{max} is the maximum evasion speed. Employing equation (11), the resultant conditional Rayleigh density function becomes

$$f_S(S|S \leq S_{max}) = \frac{S \exp\{-S^2/(2\alpha^2)\}}{\alpha^2 F(S_{max})} , \tag{12}$$

where

$$F(S_{max}) = 1 - \exp\{-S_{max}^2/(2\alpha^2)\}. \tag{13}$$

Once again, using equation (4), a conditional density function for evasion range can be obtained:

$$f_r(r|r \leq r_{max}) = \frac{r \exp\{-r^2/(2\alpha^2 t^2)\}}{\alpha^2 t^2 F(S_{max})} . \tag{14}$$

Plots of the truncated Rayleigh density function given by equation (12) are shown in figures 6a through 6d for α = 10, 15, 20, and 30, respectively. In all of these plots, the Rayleigh density was truncated at S_{max} = 35 knots. As can be seen from the plots, for small values of α, the truncation has negligible effect but, as α increases, more of the tail is dropped, causing the height near the tail to increase to keep the total area equal to one. Thus, the conditional speed density function places more weight on the evasion speeds between zero and S_{max}.

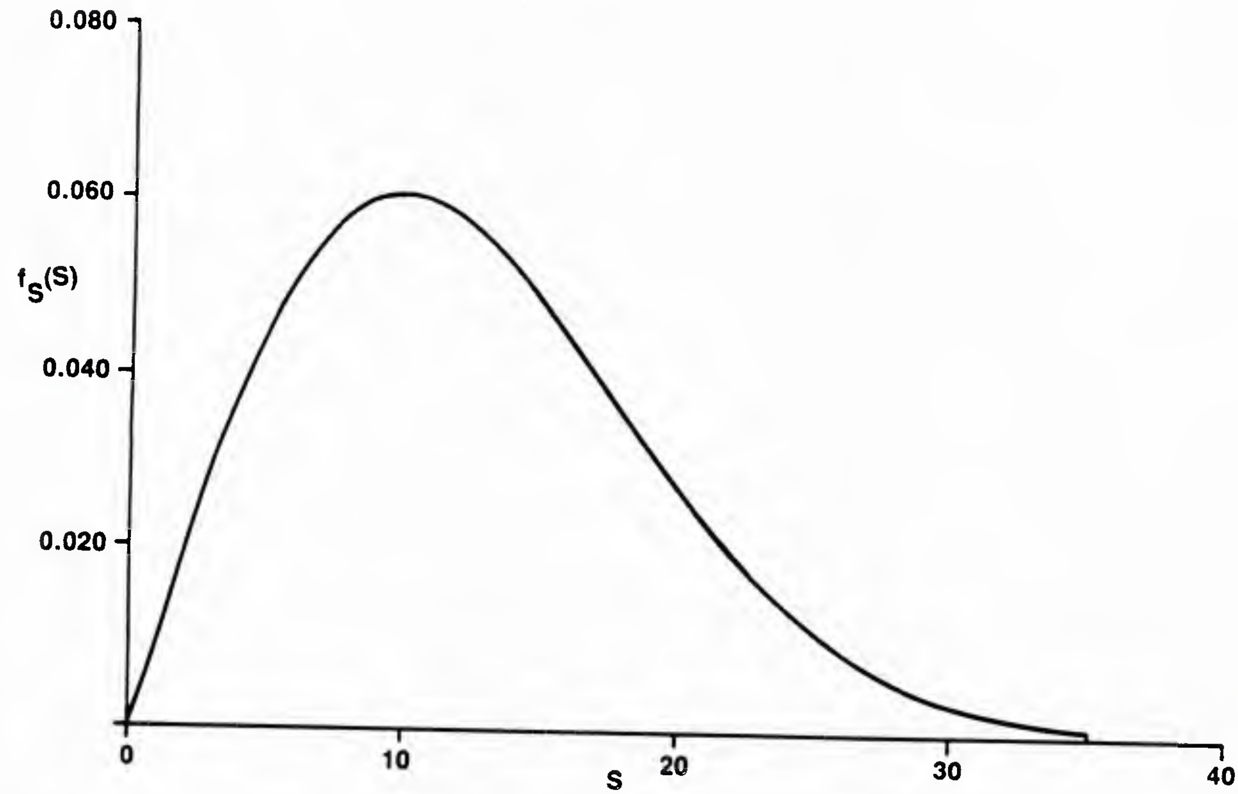

Figure 6a. Truncated Rayleigh Density Function (α = 10)

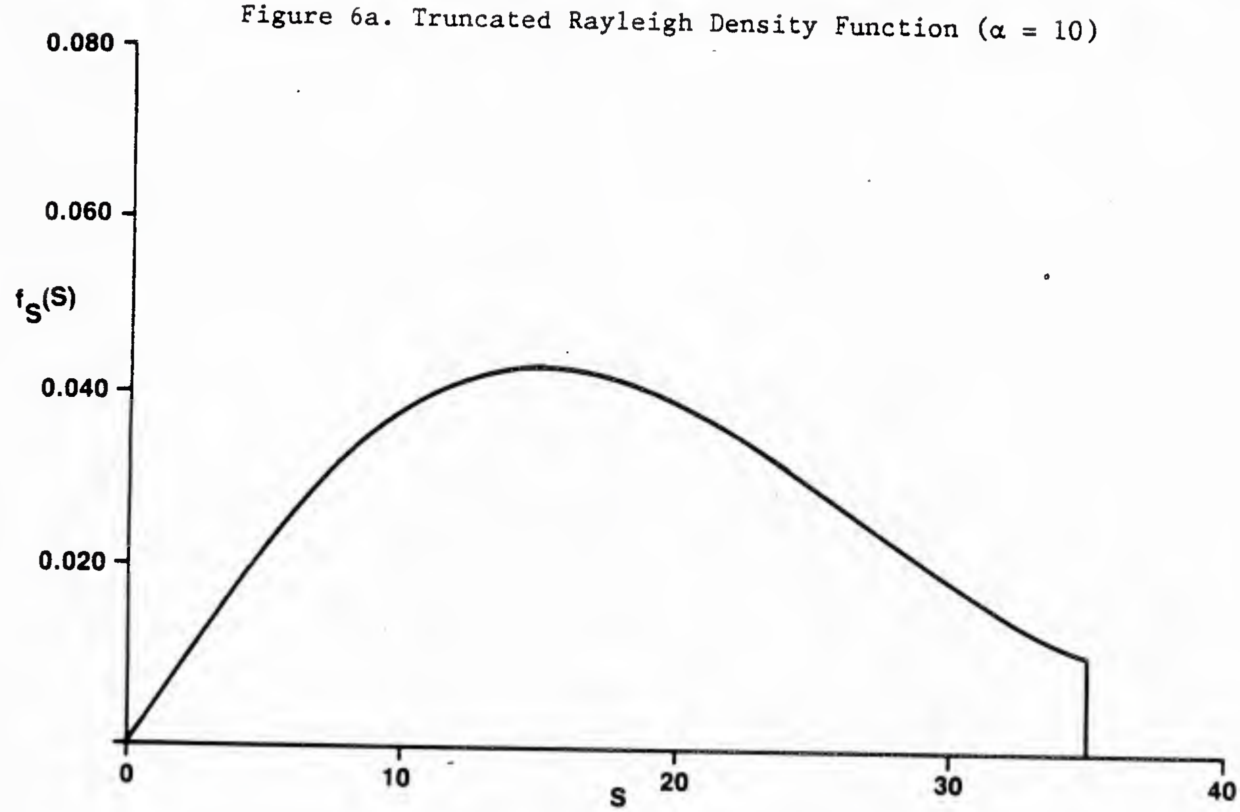

Figure 6b. Truncated Rayleigh Density Function (α = 15)

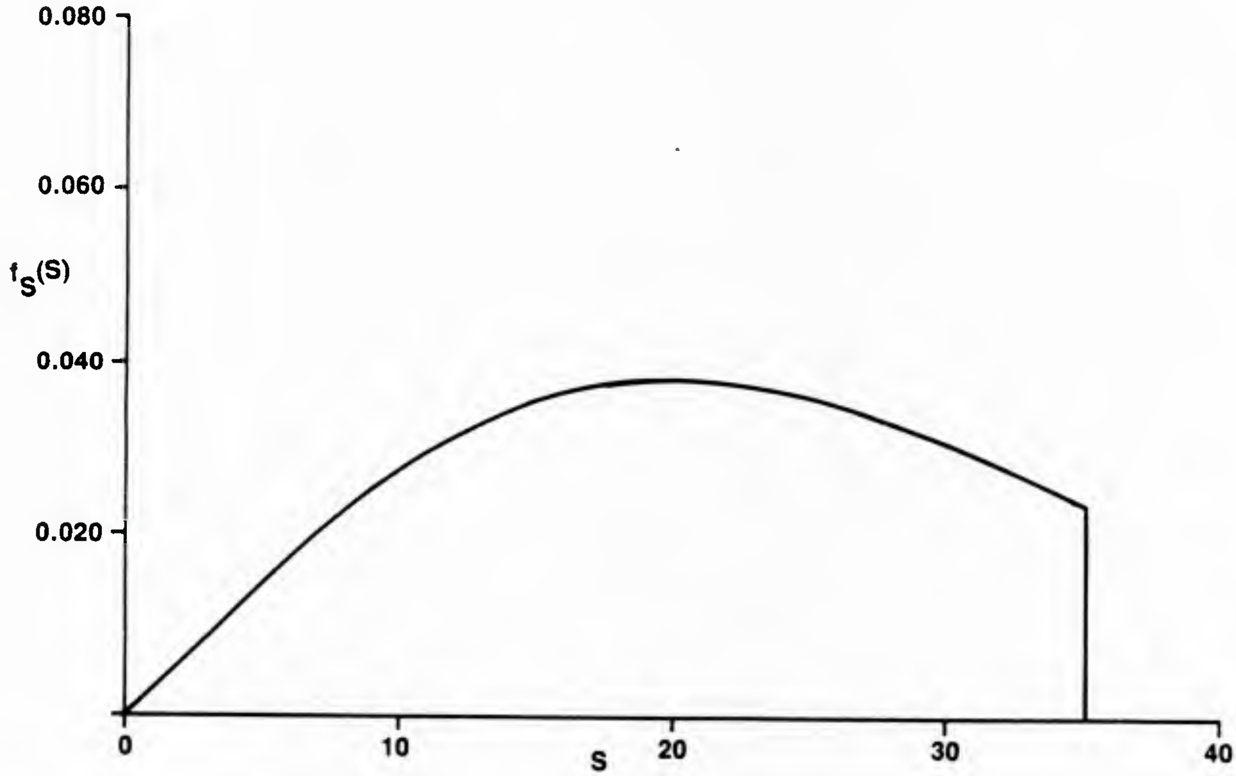

Figure 6c. Truncated Rayleigh Density Function (α = 20)

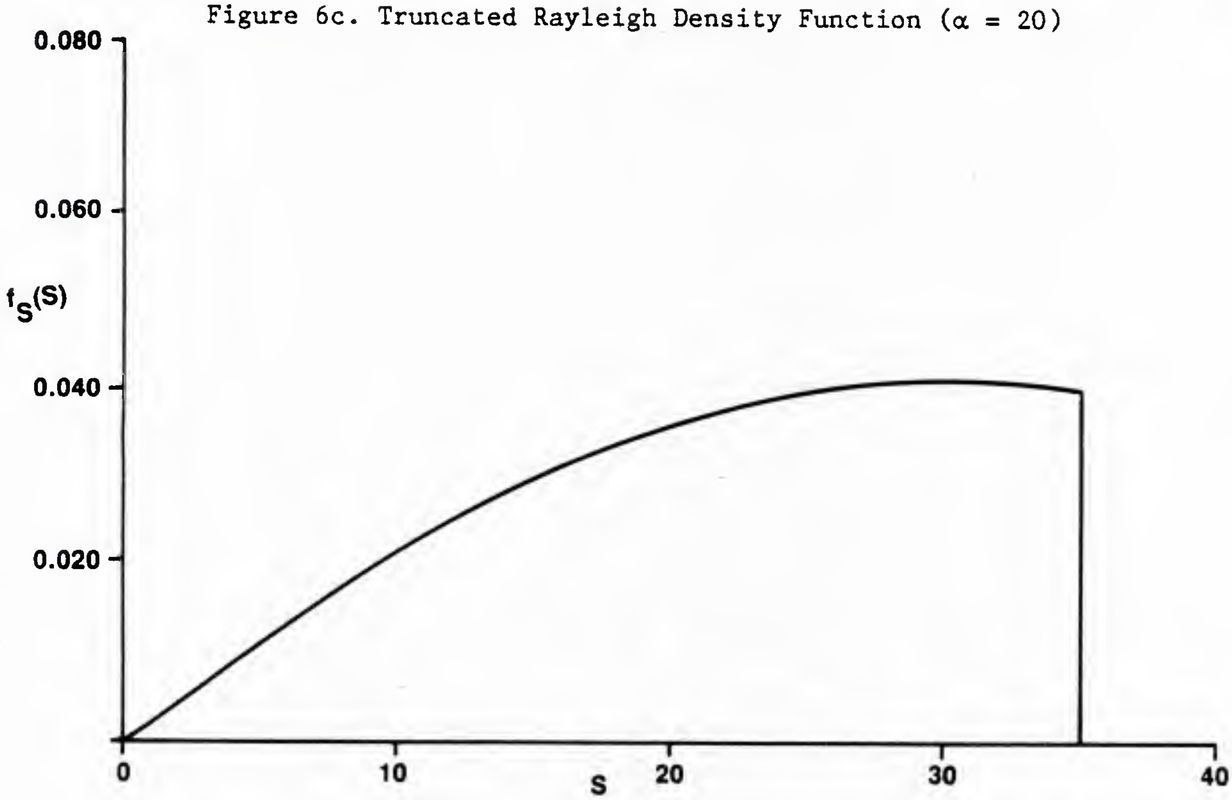

Figure 6d. Truncated Rayleigh Density Function (α = 30)

CONDITIONAL GAUSSIAN DENSITY FUNCTION

Using the uniform density function given by equation (6) to model the evasion course, the joint Gaussian probability density function for the evading target position becomes

$$f_{xy}(x,y|x^2 + y^2 \leq r_{max}^2) = \frac{\exp\{-(x^2 + y^2)/(2\alpha^2 t^2)\}}{2\pi\alpha^2 t^2 \, F(S_{max})} . \tag{15}$$

Thus, the positional density function for the target evasion region, equation (15), is a conditional Gaussian probability function, where the normalization factor is $F(S_{max})$. The error function that would normally be associated with a conditional Gaussian density function does not appear because the bounding region is a circle of radius r_{max}. Therefore,

$$F(r_{max}) = \frac{1}{2\pi\alpha^2 t^2} \iint_{x^2+y^2 \leq r_{max}^2} \exp\{-(x^2 + y^2)/(2\alpha^2 t^2)\} dxdy, \tag{16}$$

which reduces to

$$F(r_{max}) = 1 - \exp\{-r_{max}^2/(2\alpha^2 t^2)\} = F(S_{max}), \tag{17}$$

since $r_{max} = S_{max}t$.

The mean value of equation (15) is zero, and the variance is given by

$$\sigma_x^2 = \iint_{x^2+y^2 \leq r_{max}^2} x^2 f_{xy}(x,y|x^2 + y^2 \leq r_{max}^2) dxdy, \tag{18}$$

$$\sigma_y^2 = \iint_{x^2+y^2 \leq r_{max}^2} y^2 f_{xy}(x,y|x^2 + y^2 \leq r_{max}^2) dxdy. \tag{19}$$

To determine σ_x^2, equation (15) is substituted into equation (18) and a change of variables to polar coordinates is made, yielding

$$\sigma_x^2 = \frac{1}{2\pi\alpha^2 t^2 F(S_{max})} \int_0^{r_{max}} \int_0^{2\pi} r^3 \sin^3\theta \, \exp\{-r^2/(2\alpha^2 t^2)\} d\theta dr. \tag{20}$$

Integrating equation (20) with respect to θ results in

$$\sigma_x^2 = \frac{1}{2\alpha^2 t^2 F(S_{max})} \int_0^{r_{max}} r^3 \exp\{-r^3/(2\alpha^2 t^2)\}dr. \tag{21}$$

By making the substitution $z = r^2$, equation (21) becomes

$$\sigma_x^2 = \frac{1}{4\alpha^2 t^2 F(S_{max})} \int_0^{r_{max}^2} z \exp\{-z^2/(2\alpha^2 t^2)\}dz. \tag{22}$$

As shown in reference 5, equation (22) reduces to

$$\sigma_x^2 = \alpha^2 t^2 - \frac{r_{max}^2 \exp\{-r_{max}^2/(2\alpha^2 t^2)\}}{2F(S_{max})} . \tag{23}$$

Similarly, σ_y^2 can be evaluated, resulting in

$$\sigma_y^2 = \alpha^2 t^2 - \frac{r_{max}^2 \exp\{-r_{max}^2/(2\alpha^2 t^2)\}}{2F(S_{max})} . \tag{24}$$

Comparing equation (9b) with (23) and (24), it is seen that the difference between the variance of the two positional density functions -- equations (9a) and (15) -- is in the second term. Recall that the conditional probability density function was developed to keep the maximum evasion speed finite ($S_{max} < \infty$). So, if $S_{max} \to \infty$, equations (23) and (24) reduce to equation (9b); i.e., the second term goes to zero. To show this, let

$$K = \lim_{S_{max}\to\infty} \left[\frac{r_{max}^2 \exp\{-r_{max}^2/(2\alpha^2 t^2)\}}{2F(S_{max})}\right], \tag{25}$$

which can be written as

$$K = \lim_{S_{max}\to\infty} \left[\frac{S_{max}^2 t^2}{2F(S_{max}) \exp\{S_{max}^2/(2\alpha^2)\}}\right] . \tag{26}$$

Substituting equation (13) into equation (26) and employing L'Hospital's rule results in

$$K = \lim_{S_{max} \to \infty} \left[\frac{2\alpha^2 t^2}{\exp\{S_{max}^2/(2\alpha^2)\}} \right] = 0. \tag{27}$$

Figures 7a through 7d show plots of the conditional positional density function for the values of α given in figures 6a through 6d. As can be seen from these plots, for large values of α the density functions flatten out since α is directly proportional to the variance (see equations (23) and (24)). Also, the positional density function goes to zero for $r \geq r_{max}$.

Thus, the goal of obtaining a Gaussian density function to model the evasion region and still not have restrictions on the evasion speed density function has been achieved.

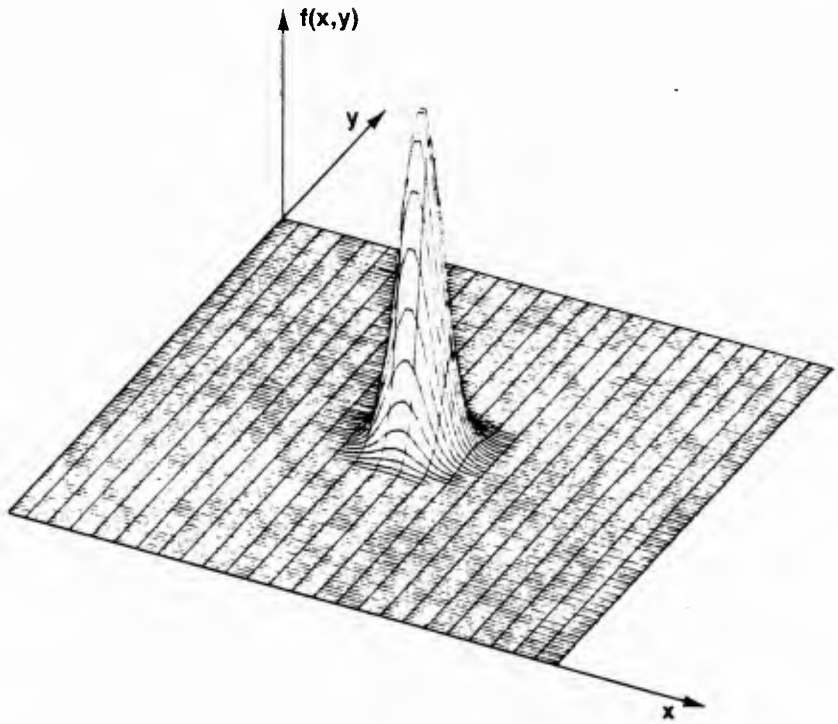

Figure 7a. Conditional Gaussian Density Function (α = 10)

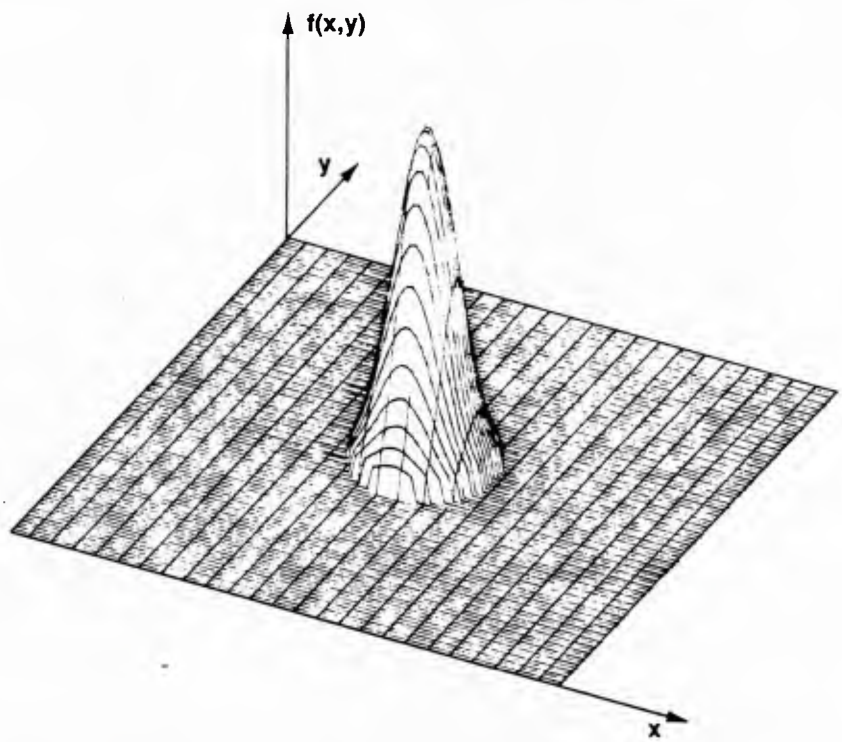

Figure 7b. Conditional Gaussian Density Function (α = 15)

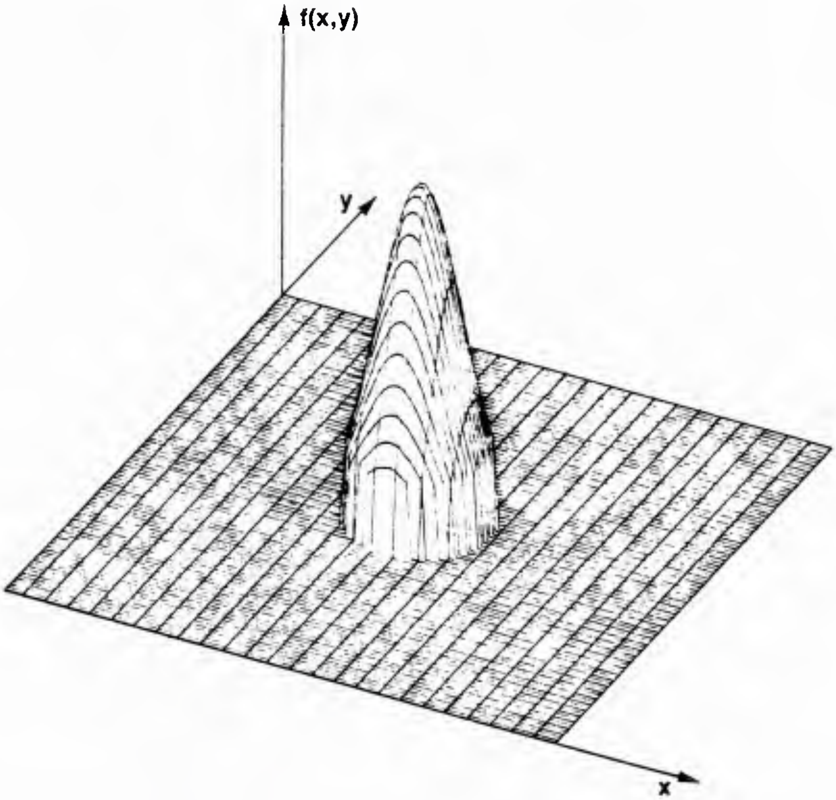

Figure 7c. Conditional Gaussian Density Function (α = 20)

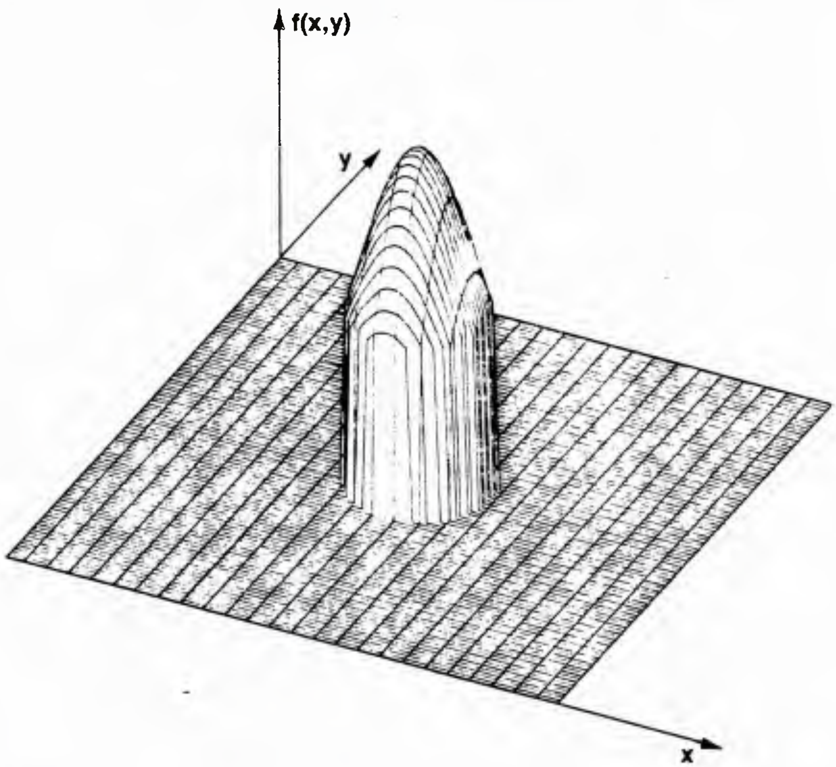

Figure 7d. Conditional Gaussian Density Function (α = 30)

APPROXIMATION TO A RAMP DENSITY FUNCTION

Figure 6d shows that as α increases, the truncated Rayleigh density function begins to resemble the ramp density function (reference 1). In fact, it can be shown that as $\alpha \rightarrow \infty$, the truncated Rayleigh density function becomes a ramp density function and, from reference 1, a ramp speed density function combined with a uniform course density function yields a uniform positional density function. To show that the truncated Rayleigh function approximates a ramp function as $\alpha \rightarrow \infty$, equation (12) can be written as

$$f_S(S|S \le S_{max}) = \frac{S \exp\{-S^2/(2\alpha^2)\}}{\alpha^2[1 - \exp\{-S_{max}^2/(2\alpha^2)\}]} \; . \tag{28}$$

Expanding the exponentials into a series yields

$$f_S(S|S \le S_{max}) = \frac{S\left[1 - \dfrac{S^2}{2\alpha^2} + \dfrac{S^4}{4\alpha^4} - \ldots\right]}{\left[\dfrac{S_{max}^2}{2} - \dfrac{S_{max}^4}{4\alpha^2} + \dfrac{S_{max}^6}{6\alpha^4} - \ldots\right]} \; . \tag{29}$$

Now take the limit as $\alpha \rightarrow \infty$, and equation (29) reduces to

$$f_S(S|S \le S_{max}) = \frac{2S}{S_{max}} \; . \tag{30}$$

Figures 8a through 8d are plots of equation (12) for $\alpha = 50$, 100, 200, and 300, respectively; figure 9 is a plot of a ramp density function. A comparison of figures 8 and 9 shows that, even for $\alpha = 100$, the truncated Rayleigh function is a good approximation of the ramp density function.

Figures 10a through 10d are plots of the positional density functions for the values of α in figures 8a through 8d. These plots show that the center flattens out as α increases and finally approximates a uniform positional density function at $\alpha = 300$.

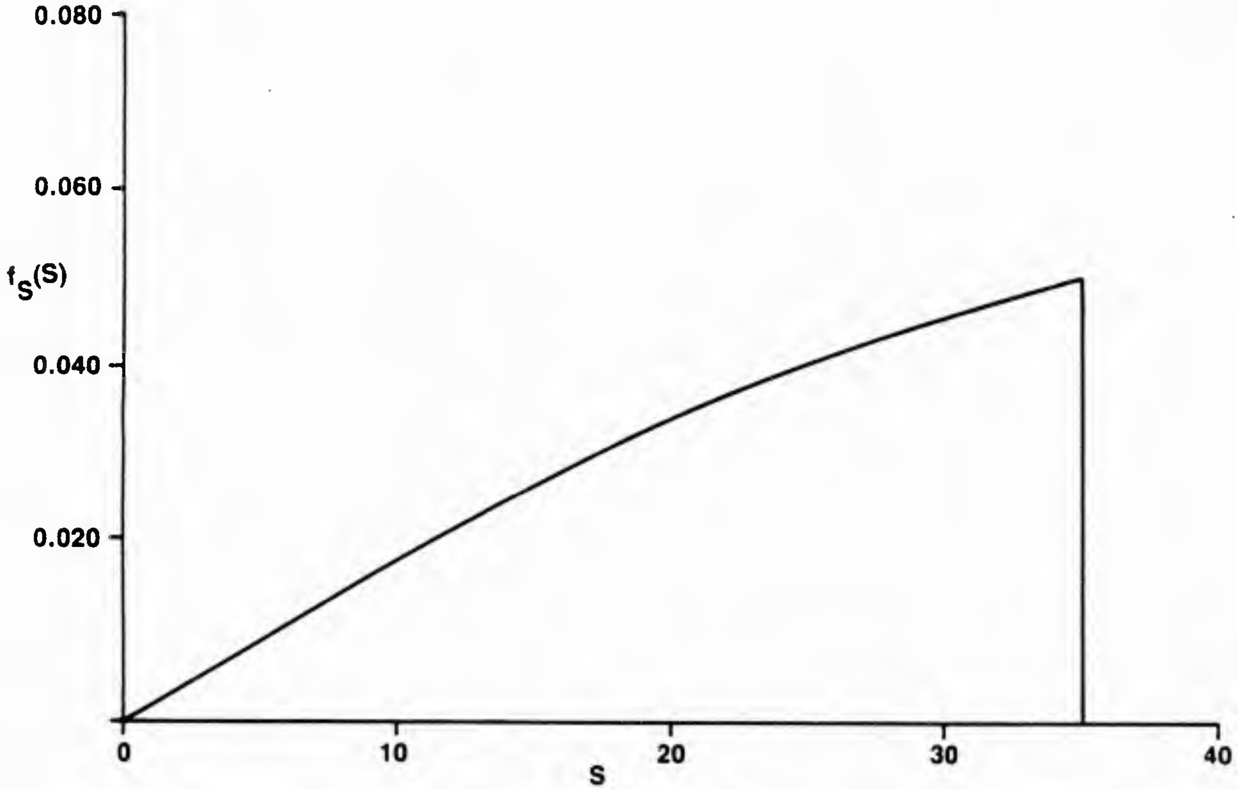

Figure 8a. Truncated Rayleigh Density Function (α = 50)

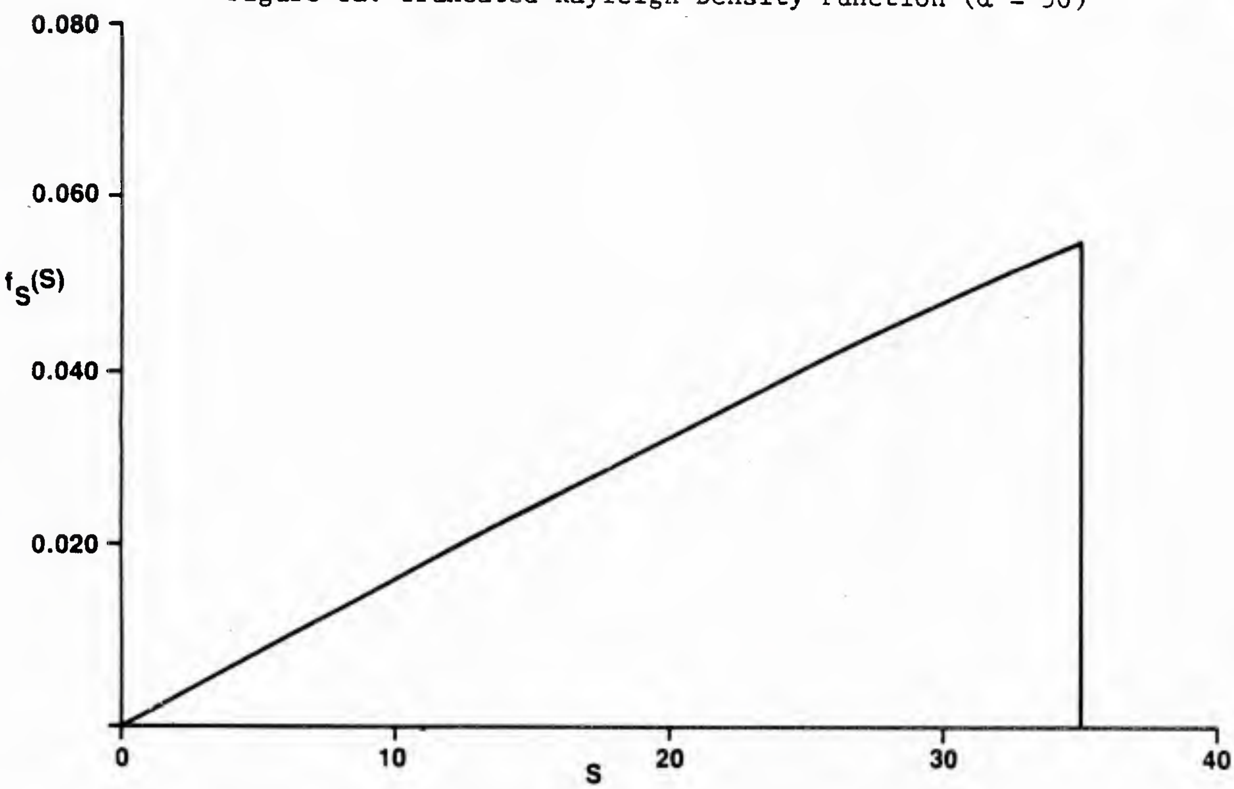

Figure 8b. Truncated Rayleigh Density Function (α = 100)

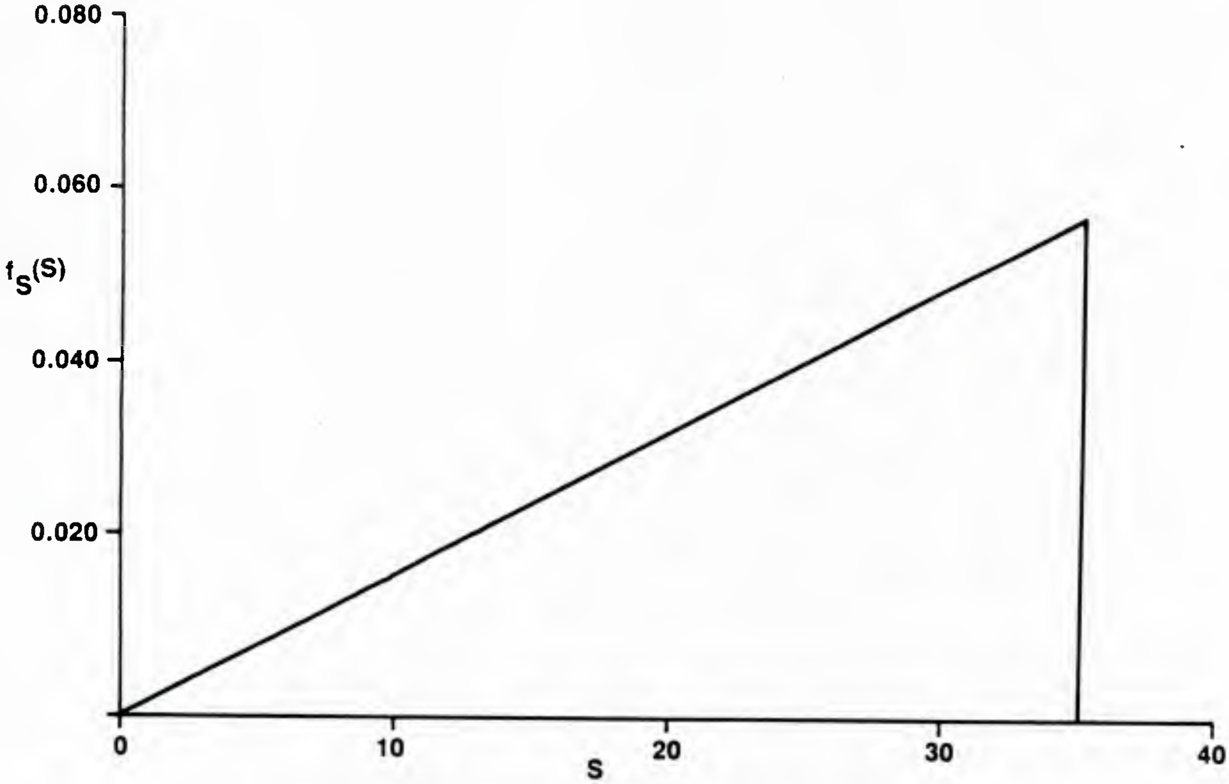

Figure 8c. Truncated Rayleigh Density Function (α = 200)

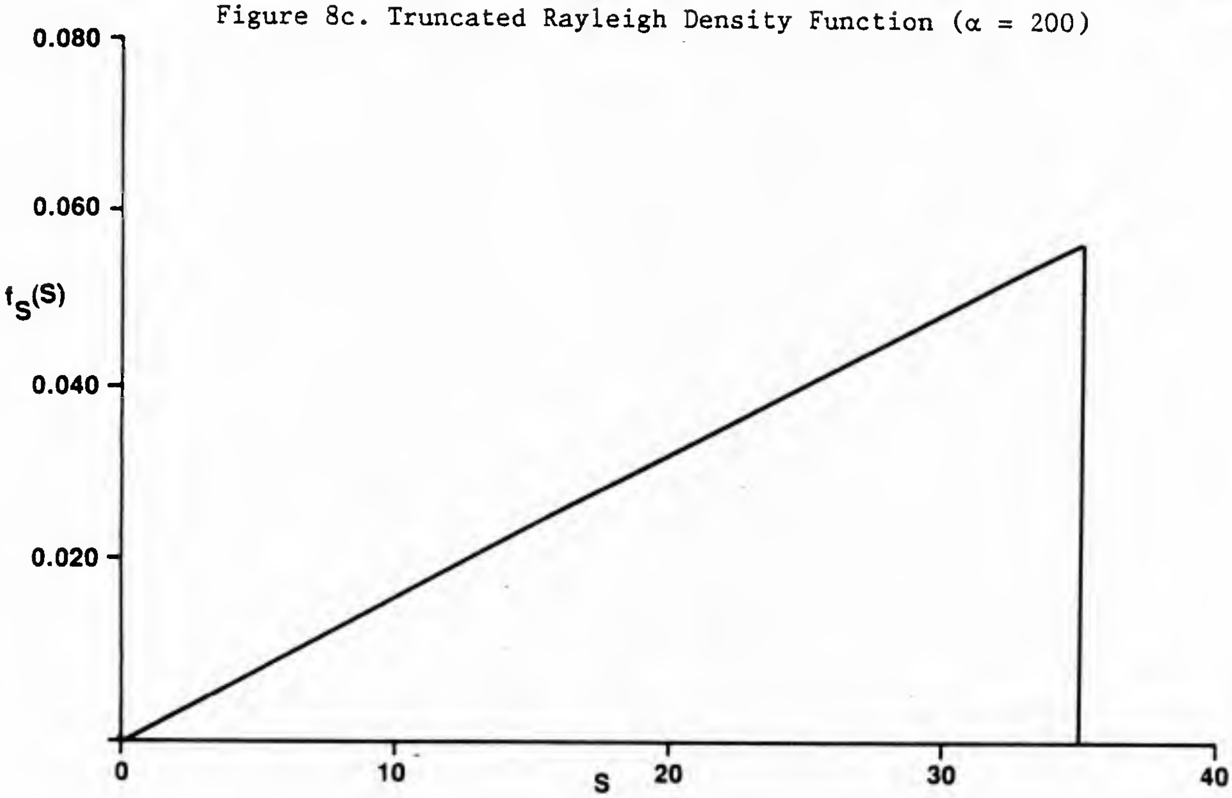

Figure 8d. Truncated Rayleigh Density Function (α = 300)

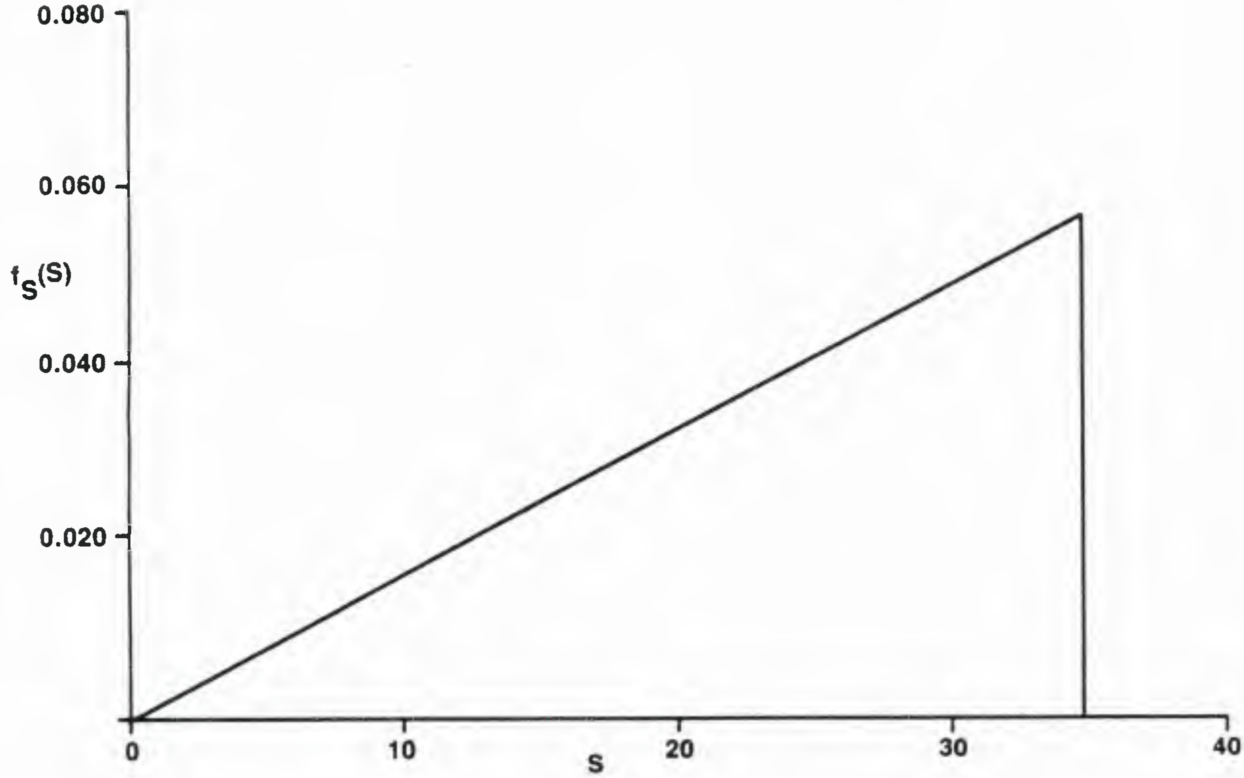

Figure 9. Ramp Density Function

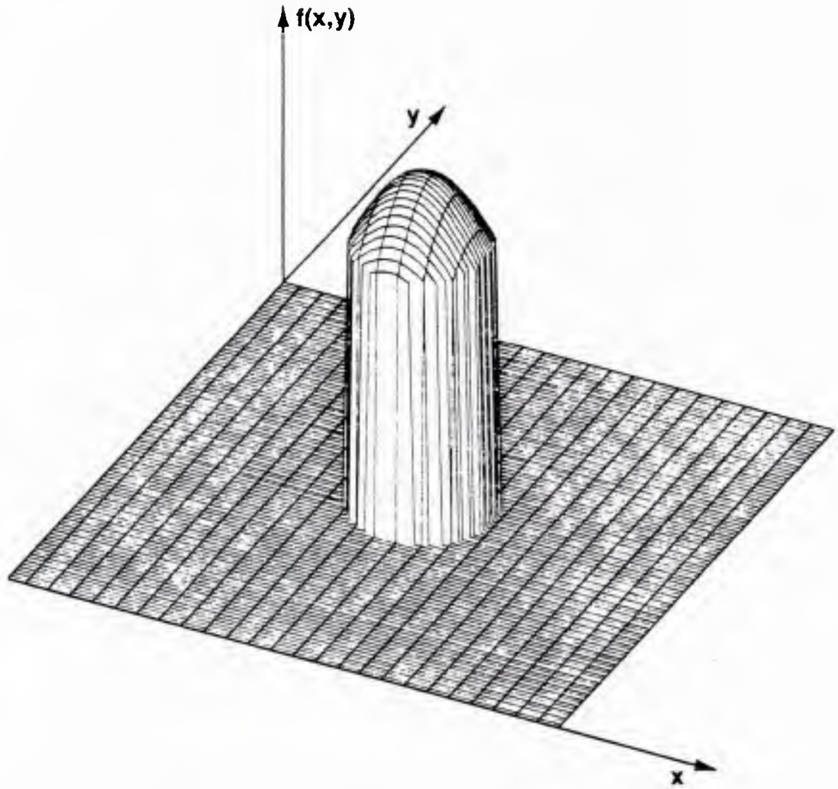

Figure 10a. Truncated Gaussian Density Function (α = 50)

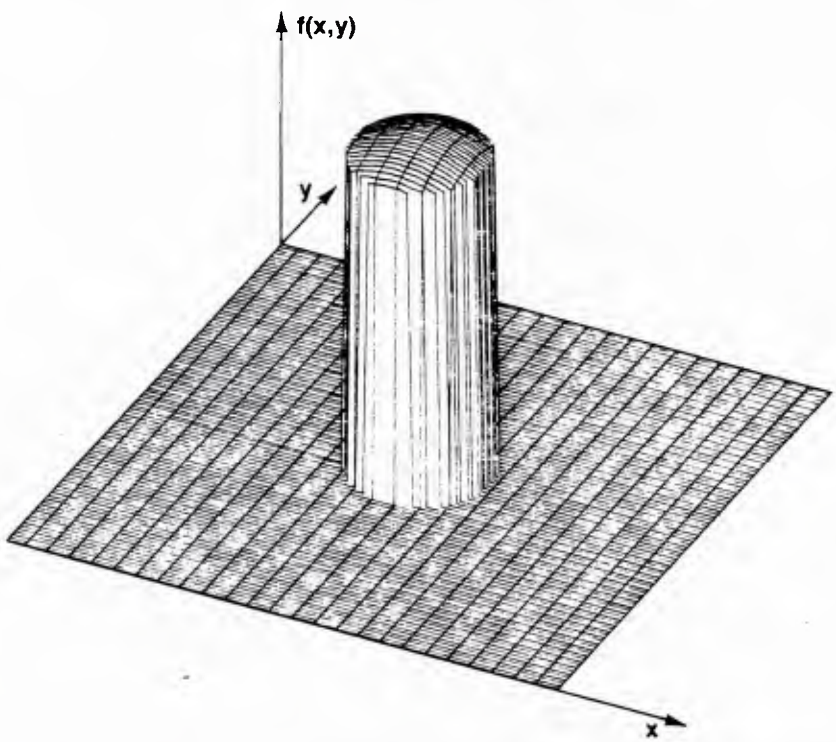

Figure 10b. Truncated Gaussian Density Function (α = 100)

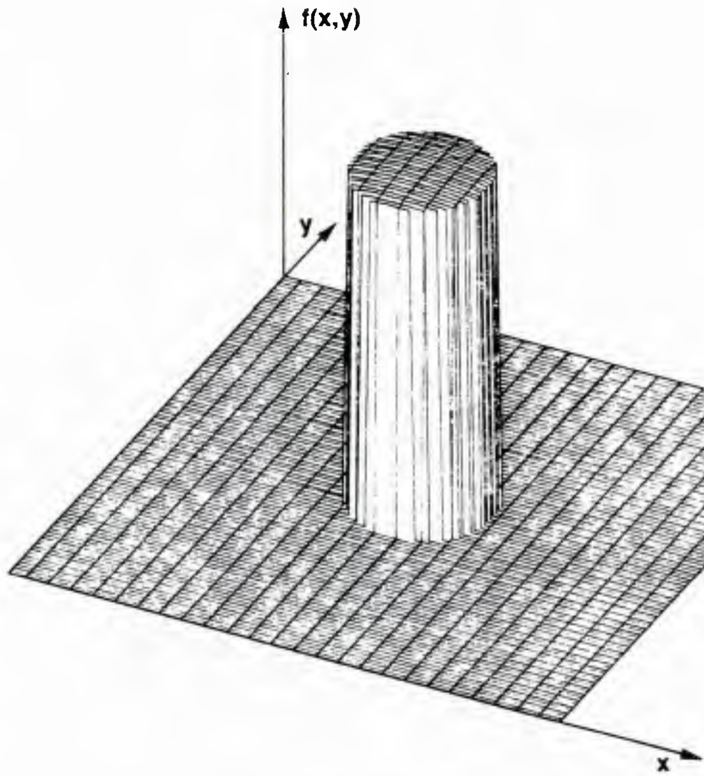

Figure 10c. Truncated Gaussian Density Function (α = 200)

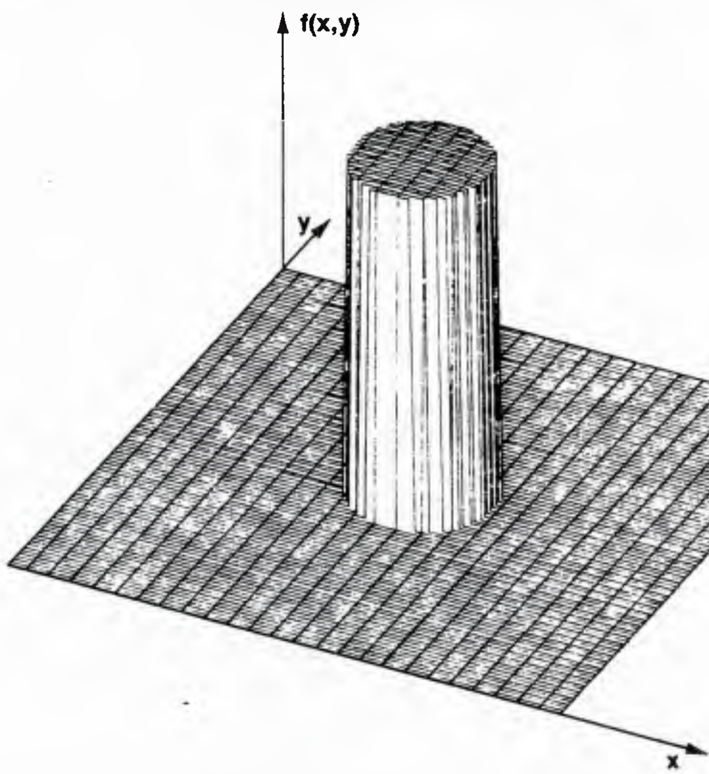

Figure 10d. Truncated Gaussian Density Function (α = 300)

III. CONCLUSIONS

This report has examined various probability density functions for modeling target evasion regions. Previous work (reference 1) employed beta density functions exclusively to achieve this goal. The work presented here uses a Rayleigh density function to model speed changes and a uniform density function to model evasion course. Unlike the beta density function with its finite limits, the Rayleigh density function has an infinite tail. Therefore, to obtain evasion speeds that are finite, the Rayleigh density function was truncated, resulting in a conditional probability density function.

The attractiveness of the Rayleigh speed/uniform course combination is that the concomitant positional density function for the target evasion region is Gaussian. The Rayleigh distribution has a parameter (α) that is analogous to the beta density function's shaping parameters; α determines the mode of the density, as well as its mean and standard deviation. This parameter also appears in the variance of the resulting Gaussian positional distribution. Consequently, the spread of the Gaussian distribution is directly related to α. The results obtained in this study also show that as $\alpha \to \infty$, a ramp distribution (beta distribution with shaping parameters a = 2, b = 1 -- see reference 1) becomes an excellent approximation of the truncated Rayleigh function.

The evasion region density function ultimately must be combined with the TMA uncertainty region, which is Gaussian distributed, to obtain online measure-of-effectiveness (MOE) algorithms. Recent investigations have shown that this is very difficult to achieve using the beta function evasion regions. However, the evasion regions modeled here are Gaussian distributed, and it is well known that a Gaussian evasion region combined with a Gaussian uncertainty region yields another Gaussian probability region. The principal advantage now is that an analytical formulation exists to perform the task of developing real-time MOEs. The only disadvantage is that target evasion course is limited to a uniform density function, and target evasion speed is limited to either a truncated Rayleigh or a ramp density function.

Future work will address the development of the interactive man-machine interface necessary to combine both a priori knowledge and heuristic aspects of the problem. These heuristic aspects (e.g., operator selection of the evasion course and speed as well as confidence levels for these selections) must be transformed into mathematical models that develop the time-varying probability density functions.

IV. REFERENCES

1. J.J. Perruzzi, R. Jackson, and L. Hoyer, "Use of Beta Density Functions in Determining Evading Target Statistics," NUSC TR 6326, Naval Underwater Systems Center, Newport, RI, December 1984. [Available To Authorized Requesters Only.]

2. E.J. Hilliard and R.F. Pinkos, "The Geometry of Escaping Targets for ASW Applications," NUSC TR 5348, Naval Underwater Systems Center, Newport, RI, June 1977. [Available To Authorized Requesters Only.]

3. W.B. Davenport and W.L. Root, An Introduction to the Theory of Random Signals and Noise, McGraw-Hill, New York, 1958.

4. D. Byrum, "ASWPK: An Algorithm to Compute Probability of Target Kill for an ASWSOW with a Nuclear Depth Bomb Payload," Report 83-QC-ASW-01, Quaternion Corp., September 1983. [Available To Authorized Requesters Only.]

5. A. Papoulis, Probability, Random Variables and Stochastic Processes, McGraw-Hill, New York, 1965.

6. I.S. Gradshteyn and I.M. Ryzhik, Tables of Integrals Series and Products, Academic Press, New York, 1965.

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